Evolution of L-fuzzy contexts associated with criteria*

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Abstract. This paper will delve deeper into the general study of the L-fuzzy contexts associated with criteria analyzing situations with a known evolution over time. These criteria may be independent or dependent on each other. We propose two different studies: to build an aggregated context looking for a simplification of the process and to keep the sequence obtaining interesting nuances over time. In both cases, aggregation operators will be used in order to address the problem. Finally, a practical example about ratings obtained by different tourist accommodations illustrates the results.

Keywords: L-fuzzy concept analysis · L-fuzzy contexts associated with criteria · WOWA operators · Choquet integrals.

1 Introduction

The L-fuzzy contexts are defined as tuples (L, X, Y, R), where L is a complete lattice, X and Y sets of objects and attributes respectively and $R \in L^{X \times Y}$ a fuzzy relation established between the sets of objects and attributes [5, 6].

As we were interested in the study of the relationship between objects and attributes from different points of view (criteria), we defined in [7] the L-fuzzy C-contexts . In [3], we also studied multivalued contexts associated with criteria. Now, we delve into the study proposing a new measure to be used in the aggregation process. We will also take into account that these L-fuzzy C-contexts can evolve with time.

We begin recovering the most important results about aggregation operators and L-fuzzy concept analysis that will be useful in the work.

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The weighted OWA operators (WOWA) were defined in [10] and combine the advantages of the OWA operators [11] and the ones of the weighted mean. These operators consider two weighting vectors, $w = (w_1, w_2, \ldots, w_n)$ for the values (operator OWA) and $p = (p_1, p_2, \ldots, p_n)$ for the experts.

Definition 1. *[10] Let p, w be weighting vectors of dimension n, $p = (p_1, p_2, \ldots, p_n)$ and $w = (w_1, w_2, \ldots, w_n)$ such that $p_i, w_i \in [0,1]$ and $\sum\limits_i p_i = \sum\limits_i w_i = 1$.*

A mapping $F_{pw} : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a Weighted Ordered Weighted Averaging (WOWA) operator of dimension n if $F_{pw}(a_1, a_2, \ldots, a_n) = \sum\limits_i \omega_i a_{\sigma(i)}$ where $\{\sigma(1), \ldots, \sigma(n)\}$ is a permutation of $\{1, \ldots, n\}$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i \in \{2, \ldots, n\}$, and the weight ω_i is defined as $\omega_i = w^*(\sum\limits_{j \leq i} p_{\sigma(j)}) - w^*(\sum\limits_{j<i} p_{\sigma(j)})$ with w^* a monotone increasing function that interpolates the points $(i/n, \sum_{j \leq i} w_j)$ together with the point (0,0). w^* is required to be a straight line when the points can be interpolated in this way.

WOWA operators have already been used in previous works [1, 3] to aggregate information in L-fuzzy contexts. In this paper the generalization of WOWA operators to Choquet integrals [8] will allow to aggregate values taking into account the existing relations among them.

Given a fuzzy measure in $\mathcal{P}(X)$, the set of parts of X, Grabisch [9] reformulated the Choquet integral as follows:

Definition 2. *The Choquet integral with respect to a fuzzy measure m can be expressed as $Ch_m(a_1 \ldots a_N) = \sum_{k=1}^{N} a_{\sigma(k)}(m(A_{\sigma(k)}) - m(A_{\sigma(k-1)}))$ where $\{\sigma(1), \ldots \sigma(N)\}$ is a permutation of $\{1, \ldots, N\}$ such that $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(N)}$, $A_{\sigma(k)} = \{a_{\sigma(j)} | j \leq k\}$ (therefore $A_{\sigma(r)} = \{a_{\sigma(1)}, \ldots, a_{\sigma(r)}\}$ when $r \geq 1$ and $A_\sigma(0) = \emptyset$).*

Besides, the derivation operators $\forall A \in L^X, \forall B \in L^Y$ are defined in [5, 6, 4]:

$$A_1(y) = \inf_{x \in X} \{I(A(x), R(x, y))\} \qquad B_2(x) = \inf_{y \in Y} \{I(B(y), R(x, y))\}$$

where I is a fuzzy implication operator defined in the lattice $(L, \leq)$.

The relevant information of the L-fuzzy context is represented by the L-fuzzy concepts $(A, A_1) \in L^X \times L^Y$, with A a fixed point of the operator φ which was defined from the derivation operators as $\varphi(A) = (A_1)_2 = A_{12}$.

Our first study of the L-fuzzy context sequences when $L = [0, 1]$ was performed in [2].

Definition 3. *An L-fuzzy context sequence is a sequence of tuples (L, X, Y, R_i), $i \in \{1, \ldots, n\}$, $n \in \mathbb{N}$, with L a complete lattice, X and Y sets of objects and attributes respectively and $R_i \in L^{X \times Y}$, for all $i \in \{1, \ldots, n\}$, a family of L-fuzzy relations between X and Y.*

We also defined in [1] an L-fuzzy relation $R_{F_{pw}}$ that aggregates the information of the different fuzzy contexts using WOWA operators.

Definition 4. *Let $(L, X, Y, R_i), i = \{1, \ldots, n\}$ be the fuzzy context sequence and F_{pw} an WOWA aggregation operator with $p = (p_1, \ldots p_n)$ and $w = (w_1, \ldots w_n)$*

such that $p_i, w_i \in [0,1]$ *and* $\sum\limits_i p_i = \sum\limits_i w_i = 1$. *Then,* $R_{F_{pw}}(x,y) = F_{pw}(R_1(x,y), \ldots R_n(x,y)) = \sum\limits_i \omega_{xy_i} R_{\sigma_{xy}(i)}(x,y)$ *where for every* (x,y) *we have* $\sigma_{xy} = \{\sigma_{xy}(1), \ldots, \sigma_{xy}(n)\}$ *a permutation of* $\{1,\ldots,n\}$ *such that* $R_{\sigma_{xy}(i-1)}(x,y) \geq R_{\sigma_{xy}(i)}(x,y)$ *for all* $i = \{2,\ldots,n\}$, *and the weighting vector* ω_{xy} *defined in Definition 1.*

The rest of the paper is organized as follows: L-fuzzy contexts associated with criteria are studied in Section 2 and in Section 3 we analyze their evolution over time. In Section 4 we apply the developed methods to a practical case. Conclusions and future work are detailed in last section.

2 *L*-fuzzy contexts associated with criteria

Sometimes, we are interested in the study of the relationship between objects and attributes from different points of view (criteria).

Definition 5. *Let be* $L = [0,1]$, *let* X, Y *and* C *be non-empty and finite sets of objects, attributes and criteria, and* $R \in L^{X\times Y}$ *an* L*-fuzzy relation. The tuple* (L, X, Y, R, C) *is said to be the* L*-fuzzy* C*-context.*

The derivation operators are defined using a fuzzy implication operator I.

Definition 6. *For every relation* $F \in L^{C\times X}$ *we define an element* F_1 *of* $L^{C\times Y}$:

$$F_1(c,y) = \inf_{x\in X}\{I(F(c,x), R(x,y))\}$$

Analogously for $G \in L^{C\times Y}$.

We proved in [7] that the L-fuzzy C-concepts are pairs $(\hat{F}, \hat{G})$ with $\hat{F} \in L^{C\times X}$ and $\hat{G} \in L^{C\times Y}$ such that applying the derivation operator to one of the relations we get the other one. The meaning of each concept is based on different criteria.

Another important point is to get an overview of every L-fuzzy C-concept. We can proceed differently depending on the dependence of the criteria.

2.1 Study with independent criteria

The starting point is the L-fuzzy C-concept $(\hat{F}, \hat{G})$ derived from $F \in L^{C\times X}$ about the degree in which every object (or attribute) verifies the different criteria. A similar process can be done taking an L-fuzzy set of attributes $G \in L^{C\times Y}$.

In order to obtain a complete information about the L-fuzzy C-concepts, we can aggregate the rows of each one of the relations that form the concept. using WOWA operators when the criteria are independent.

Definition 7. *Let be* $|C| = l$. *Given the* L*-Fuzzy* C*-concept* $(\hat{F}, \hat{G})$ *and* F_{pw} *a WOWA operator of dimension* l *associated with the weighting vectors* p *and* w, *we define the pair* $(\bar{F}, \bar{G}) \in L^X \times L^Y$ *as follows.*

$\bar{F}(x) = F_{pw}(\hat{F}(c_1,x), \hat{F}(c_2,x), \ldots \hat{F}(c_l,x))$ $\quad \bar{G}(y) = F_{pw}(\hat{G}(c_1,y), \hat{G}(c_2,y), \ldots \hat{G}(c_l,y))$

Using WOWA operators we can establish different nuances in our study.

2.2 Study with dependent criteria

We have defined the L-fuzzy C-contexts as a model to represent the relationship between objects and attributes from different points of view (criteria).

If the criteria are dependent then a group of them can give a better result if they are combined instead of being treated in isolation.

With this aim, we proposed in [3] the use of Choquet integrals to aggregate the values of the L-fuzzy C-concept $(\hat{F}, \hat{G})$ derived from $F \in L^{C\times X}$.

The actions are described in the following algorithm.

Algorithm 1 Criteria Aggregation Process (CAP)

Input: The L-fuzzy C-concept $(\hat{F}, \hat{G})$ derived from $F \in L^{C\times X}$.
Output: Aggregated pair $(\bar{F}^{\hat{\alpha}}, \bar{G}^{\hat{\alpha}}) \in L^X \times L^Y$.

1: For every $c_k, k \in \{1, \ldots, l\}$ we obtain its derived L-fuzzy concept $\mathcal{C}_\mathbf{k}$ in the L-fuzzy context (L, C, X, F).
2: For every $\mathcal{C}_\mathbf{k}, k \in \{1, \ldots, l\}$ and given $0 < \alpha \leq 1$, we define the set $T^\alpha_{\mathcal{C}_\mathbf{k}}$ of α-objects associated with $\mathcal{C}_\mathbf{k}$ as $T^\alpha_{\mathcal{C}_\mathbf{k}} = \{c_i \in C \mid memb(c_i, \mathcal{C}_\mathbf{k}) \geq \alpha\}$.
3: Select the maximum value $\hat{\alpha}$ such that all the sets $T^{\hat{\alpha}}_{\mathcal{C}_\mathbf{k}}$ are connected.
4: Aggregate the rows of the L-Fuzzy C-concept $(\hat{F}, \hat{G})$ associated with the different $c_k, k \in \{1, \ldots, l\}$ using the Choquet integral associated with m. The result is a pair of L-fuzzy sets $(\bar{F}^{\hat{\alpha}}, \bar{G}^{\hat{\alpha}}) \in L^X \times L^Y$:
$$\bar{F}^{\hat{\alpha}}(x) = Ch_m(F(c_1, x), F(c_2, x), \ldots F(c_l, x))$$
$$\bar{G}^{\hat{\alpha}}(y) = Ch_m(G(c_1, y), G(c_2, y), \ldots G(c_l, y))$$
with Ch_m the Choquet integral associated to m and $c_k \in C, k \in \{1, \ldots, l\}$.

The pair $(\bar{F}^{\hat{\alpha}}, \bar{G}^{\hat{\alpha}}) \in L^X \times L^Y$ is not necessarily an L-fuzzy concept of the L-fuzzy context (L, X, Y, R).

We propose in this work a new measure to simplify the calculations.

For any $A \subseteq E$, we take the one defined by the number of sets $T^{\hat{\alpha}}(\mathcal{C}_\mathbf{k})$ to which the elements of A belong:

$m(A) = Card(T_{\hat{\alpha}})/l$, where $T_{\hat{\alpha}} = \{T^{\hat{\alpha}}(\mathcal{C}_\mathbf{k}), k \leq l \mid A \bigcap T^{\hat{\alpha}}(\mathcal{C}_\mathbf{k}) \neq \emptyset\}$.

3 Study of the evolution of contexts with criteria

Sometimes, the relationship between objects and attributes from different points of view (criteria) may be different with the passage of time.

Definition 8. *An L-fuzzy context sequence associated with criteria is a sequence of tuples (L, X, Y, R_i, C), $i \in \{1, \ldots, n\}$, $n \in \mathbb{N}$, with $L = [0, 1]$ a complete lattice, X, Y and C sets of objects, attributes and criteria respectively and $R_i \in L^{X\times Y}$, for all $i \in \{1, \ldots, n\}$, a family of L-fuzzy relations between X and Y.*

If can consider two different ways to study of the evolution of the relationship between the objects X and the attributes Y taking into account the criteria

C. The first one is to build a new context aggregating the L-fuzzy C-context sequence and the second one to keep the sequence to analyze different moments in the study. In the second case, for every L-fuzzy C-context $(L, X, Y, R_i, C), i \in \{1, \ldots, n\}$ of the sequence, we can obtain the L-fuzzy C-concepts that represent the relationship between the objects and the attributes in a fixed moment, taking into account the different criteria.

In general, we will decide in what moment we will aggregate the values looking for a simplification of the process at the cost of losing interesting nuances such as the evolution of the relationship over time or the differences between the different criteria. Let us see it in the following sections.

3.1 Aggregation of the L-fuzzy contexts associated with criteria

To aggregate the L-fuzzy C-contexts of the sequence, we will use techniques of L-fuzzy contexts sequences [2].

Let $(L, X, Y, R_i, C), i = \{1, \ldots, n\}$, be the L-fuzzy C-context sequence and F_{pw} an WOWA aggregation operator with $p = (p_1, \ldots p_n)$ and $w = (w_1, \ldots w_n)$ such that $p_i, w_i \in [0, 1]$ and $\sum\limits_i p_i = \sum\limits_i w_i = 1$. Then,

$$R_{F_{pw}}(x, y) = F_{pw}(R_1(x, y), \ldots R_n(x, y)) = \sum_i \omega_{xy\,i} R_{\sigma_{xy}(i)}(x, y)$$

where for every (x, y) we have $\sigma_{xy} = \{\sigma_{xy}(1), \ldots, \sigma_{xy}(n)\}$ a permutation of $\{1, \ldots, n\}$ such that $R_{\sigma_{xy}(i-1)}(x, y) \geq R_{\sigma_{xy}(i)}(x, y)$ for all $i = \{2, \ldots, n\}$, and the weighting vector ω_{xy} defined in Definition 1.

We have now an L-fuzzy C-context $(L, X, Y, R_{F_{pw}}, C)$. Next, we will start from a relation $F \in L^{C \times X}$ that represents the valuations of the different objects X regarding the different criteria and we will calculate its L-fuzzy C-concept $(\hat{F}, \hat{G}) \in L^{C \times X} \times L^{C \times Y}$. The same process can be done from $G \in L^{C \times Y}$.

Finally, a pair $(\bar{F}, \bar{G}) \in L^X \times L^Y$ can be obtained by aggregating the values of the L-fuzzy C-concept rows associated with the different criteria, using WOWA operators or Choquet integrals, depending on the independence or dependence of the criteria. This is the procedure that we use when we want an overview of our L-fuzzy C-concept obtained from $F \in L^{C \times X}$.

3.2 Study of L-fuzzy contexts associated with criteria and time

In the previous subsection we have aggregated the different L-fuzzy contexts, losing information relative to the different moments of the study. For this reason, it is necessary a complementary study in order to maintain the information associated with each value of the sequence until the end of the process.

Let us start with the L-fuzzy context sequence associated with criteria and the L-fuzzy relation $F \in L^{C \times X}, (G \in L^{C \times Y})$ representing the valuations of the different objects X (attributes Y) with respect to the different criteria.

We can calculate the L-fuzzy C-concepts $(\hat{F}_i, \hat{G}_i), i \in \{1, \ldots, n\}$ for every L-fuzzy C-context of the sequence.

Definition 9. *For every criteria* $c_k, k \in \{1, \ldots, l\}$, *we define* $(\hat{F}_i^{c_k}, \hat{G}_i^{c_k}) \in L^X \times L^Y$ *as* $\hat{F}_i^{c_k}(x) = \hat{F}_i(c_k, x)$, *for all* $x \in X$ *and* $\hat{G}_i^{c_k}(y) = \hat{G}_i(c_k, y)$, *for all* $y \in Y$.

The previous pairs are L-fuzzy concepts (see [7]) associated with the criteria c_k in the L-fuzzy contexts $(L, X, Y, R_i), i \in \{1, \ldots, n\}$. Now we have two options:

- Aggregate the criteria: We get a sequence of pairs of L-fuzzy sets $(\bar{F}_i, \bar{G}_i) \in L^X \times L^Y, i \in \{1, \ldots, n\}$. In case of not taking into account the dependence of the criteria, the Choquet integrals that we will use will be WOWA operators.
- Work with the L-fuzzy concepts $(\hat{F}_i^{c_k}, \hat{G}_i^{c_k}), i \in \{1, \ldots, n\}$ associated with each criterion $c_k, k \in \{1, \ldots, l\}$.

4 Practical example

To show the application of the results, we have taken data from hotel occupancy surveys from *INE (the Spanish National Statistics Institute).* The data is represented by means of an L-fuzzy context sequence $(L, X, Y, R_i, C), i \in \{1, \ldots, 5\}$ that collects the average stay of clients in tourist establishments (X) in some of regions (Y) over a period of time.

We want to study the types of establishments and regions where there has been a longer average stay and also in those cases where it has been maintained or increased over time. To do this, we will consider certain criteria C that can be associated with profiles of potential users. In this case, the criteria are $C = \{$economical trip, cultural trip, trip to a rural environment $\}$. We will assume initially that these are independent criteria to analyze their dependence later.

In this L-fuzzy context, $X = \{$Hotels, Camp sites, Tourist Apartments, Rural Accommodation $\}$, $Y = \{$Andalusia, Catalonia, Navarre, Basque Country$\}$ and the following relations correspond with the average occupancy rate between March to July 2018. We have normalized the values to $L = [0, 1]$. (see Table 1).

R_1	y_1	y_2	y_3	y_4
x_1	0.4	0.4	0.3	0.3
x_2	0.8	0.5	0.8	0.4
x_3	0.7	0.8	0.6	0.6
x_4	0.5	0.4	0.5	0.4

R_2	y_1	y_2	y_3	y_4
x_1	0.4	0.4	0.3	0.3
x_2	0.6	0.6	0.5	0.3
x_3	0.6	0.8	0.5	0.6
x_4	0.5	0.3	0.4	0.4

$\ldots$

R_5	y_1	y_2	y_3	y_4
x_1	0.5	0.5	0.3	0.3
x_2	0.6	1	0.5	0.5
x_3	0.8	1	0.5	0.7
x_4	0.7	0.6	0.7	0.4

Table 1. L-fuzzy context sequence.

First, we are going to define a relation $R_{F_{pw}}$ that summarizes the sequence using WOWA operators. Then, we can start from $F \in L^{C \times X}$, (see Table 2) which represents the valuation of the different tourist accommodations taking into account the different criteria (economical trip, cultural trip, trip to a rural environment). We obtain the L-fuzzy C-concept $(\hat{F}, \hat{G})$ in the L-fuzzy context associated with relation $R_{F_{pw}}$.

F	x_1	x_2	x_3	x_4
c_1	0.3	1	1	0.8
c_2	1	0.5	1	0.5
c_3	0	0	0	1

$\rightarrow$

$\hat{F}$	x_1	x_2	x_3	x_4
c_1	0.8	1	1	0.8
c_2	1	1	1	1
c_3	0.7	0.9	0.9	1

$\hat{G}$	y_1	y_2	y_3	y_4
c_1	0.6	0.7	0.5	0.4
c_2	0.5	0.5	0.3	0.3
c_3	0.6	0.5	0.6	0.5

Table 2. L-fuzzy concept associated with criteria.

We can now draw conclusions from the period of time analyzed according to the different profiles of travelers associated with the criteria.

a) Among the people traveling with little money (c_1), the highest average stay is in the camp sites and tourist apartments (x_2 e x_3) of Catalonia (y_2) and to a lesser extent, Andalusia (y_1).
b) For those who make cultural trips (c_2), establishments in Andalusia and Catalonia have relevant average stays.
c) In the case of the traveler profile seeking peace in rural areas (c_3), Andalusia and Navarre (y_1 e y_3) are regions with longer stays.

If the criteria are independent, we can obtain a summary by aggregating the rows associated to the different criteria through a WOWA operator:
$(\bar{F},\bar{G}) = (\{x_1/0.88, x_2/0.98, x_3/0.98, x_4/0.97\}, \{y_1/0.58, y_2/0.60, y_3/0.52, y_3/0.43\})$

We can therefore interpret this pair saying that *for the profiles of travelers established by the criteria, in the camp sites, tourist apartments and, to a lesser extent, in the rural tourism accommodation in Catalonia is where we find longer average stays, followed by Andalusia.*

The result would not be the same if we considered the dependence of criteria. We can apply the *Criteria Aggregation Process (CAP)* described in section 2.2. We will start with $F \in L^{C\times X}$, which represents the valuation of the different establishments according to the criteria.

For every $c_k, k \in \{1,\ldots,3\}$ we obtain its derived L-fuzzy concept $\mathcal{C}_\mathbf{k}$ in the L-fuzzy context (L, C, X, F). The result is:

$\mathcal{C}_\mathbf{1} = (\{c_1/1, c_2/0.5, c_3/0\}, \{x_1/0.3, x_2/1, x_3/1, x_4/0.8\})$
$\mathcal{C}_\mathbf{2} = (\{c_1/0.3, c_2/1, c_3/0\}, \{x_1/1, x_2/0.5, x_3/1, x_4/0.5\})$
$\mathcal{C}_\mathbf{3} = (\{c_1/0.8, c_2/0.5, c_3/1\}, \{x_1/0, x_2/0, x_3/0, x_4/1\})$

Then $\hat{\alpha} = 0.5$ and the resulting measure: $m(c_1) = 2/3, m(c_2) = 1, m(c_3) = 1/3, m(c_1,c_2) = m(c_2,c_3) = 1, m(c_1,c_3) = 2/3, m(C) = 1$.

Finally, the result of aggregating with Choquet integral is:
$(\bar{F},\bar{G}) = (\{x_1/1, x_2/1, x_3/1, x_4/1\}, \{y_1/0.57, y_2/0.63, y_3/0.47 y_3/0.4\})$

If we take into account the dependence of the criteria obtained from F, the longest average occupancy for the clients of the profiles analyzed are mainly in the region of Catalonia and also in Andalusia. There are no differences between the different establishments.

We can also work with each one of the contexts of the sequence and obtain similar information starting from $F \in L^{C\times X}$ for each of the months of the study:

$(\bar{F_1},\bar{G_1}) = (\{x_1/0.9, x_2/1, x_3/1, x_4/1\}, \{y_1/0.6, y_2/0.5, y_3/0.5, y_4/0.4\})$
$(\bar{F_2},\bar{G_2}) = (\{x_1/0.9, x_2/1, x_3/1, x_4/0.9\}, \{y_1/0.5, y_2/0.4, y_3/0.4, y_4/0.4\})$
$(\bar{F_3},\bar{G_3}) = (\{x_1/0.9, x_2/1, x_3/1, x_4/0.9\}, \{y_1/0.5, y_2/0.5, y_3/0.4, y_4/0.3\})$

$(\bar{F_4}, \bar{G_4}) = (\{x_1/1, x_2/1, x_3/1, x_4/0.9\}, \{y_1/0.6, y_2/0.5, y_3/0.4, y_4/0.4\})$
$(\bar{F_5}, \bar{G_5}) = (\{x_1/0.8, x_2/1, x_3/1, x_4/1\}, \{y_1/0.6, y_2/0.7, y_3/0.6, y_4/0.5\})$

Looking at these pairs we can extract some important conclusions taking into account the profiles of clients represented by the criteria:

a) In the month of March the longest stays are in Andalusia (y_1) and in camp sites, tourist apartments and rural tourism accommodations (x_2, x_3 and x_4).
b) Tourist camp sites and apartments (x_2 and x_3) in Andalusia (y_1) are the establishments that stand out in April. In May also in Catalonia (y_2).
c) In the month of June, hotels, camp sites and tourist apartments (x_1, x_2 and x_3) stand out for their average stays in Andalusia (y_1).
d) In July the camp sites, tourist apartments and rural tourism accommodation (x_2, x_3 and x_4) are those that have longer stays especially in Catalonia (y_2).

5 Conclusions and future work

The main contribution of the paper to the literature is the analysis of different ways of complementary study for contexts associated with criteria for which its evolution over time is known. The nature of the criteria and our interest of study will help us to select the most appropriate one. Specifically, we have proposed two different studies: build an aggregated context looking for a simplification of the process or keep the sequence obtaining interesting nuances such as the evolution of the relationship over time or the differences between the criteria.

In a future work, we will study sets of criteria where some of their elements are dependent on each other and others are not.

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